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Analysis of cosmic ray data in terms of models of high energy interactions

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Abstract. A variety of cosmic ray data are analysed from the point of view of determining the effective multiplicity of charged pions produced in the interactions of the primary cosmic rays with air nuclei. If intranuclear cascading is small and the primaries are mainly protons then the multiplicity is higher than that given by the 'scaling' hypothesis, which has been shown to have a measure of applicability at ISR energies. Agreement with scaling can be restored if the mean mass of the primary particles increases dramatically above a few times 10^{14} eV but such a change seems very unlikely both in view of various extensive air shower phenomena and because of astrophysical considerations.

1. Introduction

The interrelation of the various secondary components of the cosmic radiation and their relationship to the primary spectrum has been studied for many years and information has been gained about a variety of nuclear physical parameters. As examples of the value of this work one can instance the derivation of the mean inelasticity in nucleon–air nucleus collisions ($\langle K\rangle \sim 0{\cdot}5$), the comparative lack of importance of kaon production even at proton energies as high as 10^{13} eV, the unlikelihood of drastic changes of nucleon–nucleon cross sections to energies as high as 10^{19} eV and the very slow change with primary energy of the mean transverse momentum of secondaries produced in such collisions (a good summary of the cosmic ray data of significance for high energy physics has been given recently by Feinberg 1972a, b).

In a number of papers, starting with de Beer *et al* (1966), we have examined a variety of characteristics of extensive air showers (EAS) and endeavoured to interpret them in terms of nuclear physical and astrophysical parameters. Our philosophy has been to take as a datum a 'conservative model' of high energy interactions and to use it to predict various EAS characteristics and to relax the model correspondingly to fit the experimental data. The main features of the model were a secondary pion multiplicity varying with primary energy, E_p, as E_p^α, the exponential secondary pion energy spectrum $N(E_\pi)$ proposed by Cocconi *et al* (1961, to be referred to as CKP), namely $N(E_\pi) = (A/T)\exp(-E_\pi/T)$ in the forward cone, where A is the forward multiplicity and T is the mean forward pion energy, and constant nucleon–nucleus and pion–nucleus cross sections and inelasticities. The variable parameters were α, the mean transverse momentum $\langle p_t\rangle$ and, in some areas of primary energy, the mean mass of the particles. The conclusions arrived at (see, for example Adcock *et al* 1971) were that $\alpha \simeq \frac{1}{4}$, $\langle p_t\rangle$ increases very slowly from about 0·4 GeV/c at $E_p \simeq 10^{12}$ eV and that the primary mass is rather close to that of the proton.

The main features of the model (CKP energy spectrum, etc) came from empirical fits to the accelerator data available in the mid 1960's, namely, to $E_p \simeq 3\times10^{10}$ eV. More recently, the intersecting storage rings (ISR) have been brought into operation at CERN and new evidence has been provided for the results of p–p collisions at effective primary energies up to $1{\cdot}5\times10^{12}$ eV (see, for example the papers by Morrison 1972, and Jacob 1972 for summaries). These results have demonstrated a somewhat different pion energy spectrum from that given by CKP and it is necessary to see their implications for the analysis of cosmic ray data in the common energy region, that is, below about $1{\cdot}5\times10^{12}$ eV. Above this value the cosmic ray data in turn can be examined to see to what extent the behaviour exhibited at ISR energies continues to these higher energies.

A most interesting feature of the ISR results is that some, at least, are consistent with predictions from the 'scaling' hypothesis of Feynman (1969) and that of limiting fragmentation (Benecke *et al* 1969). A prominent feature is that the form of the distribution of pion energy in interactions taken inclusively, expressed in terms of the fractional energy x taken by a pion, appears to be constant in p–p collisions, for $x \gtrsim 0{\cdot}1$, over a wide range of proton energy ($\sim10^{10}$–10^{12} eV). This feature is exactly that expected from the scaling hypothesis. Also predicted, and observed, is a near constancy of $\langle p_t \rangle$.

Important predictions of the scaling 'model' are the logarithmic increase in the total multiplicity of secondary pions and the fact that the characteristics of the energetic forward particles should be determined by the nature of the incident particle and not the struck particle. In fact this multiplicity law is not observed in going from 2×10^{10} eV to ISR energies (figure 1) and it has been argued that complete scaling for pions does not set in until about 10^{12} eV. The distinction between incident particle and struck particle cannot be checked in the accelerator p–p experiments but it has relevance in the cosmic ray situation where collisions involving p–p, n–p, p–n, etc are encountered.

It should be remarked at this stage that, in addition to the non-observation of the logarithmic multiplicity dependence at accelerator energies, other features, too, do not accord with scaling predictions, notably; the energy spectra of protons, antiprotons and kaons.

The scaling behaviour appears to be of fundamental importance in that it relates to the structure of the nucleon and an examination of the extent to which it may be valid at cosmic ray energies is of considerable interest. Previous notes by the present authors (Wdowczyk and Wolfendale 1972, 1973) gave brief consideration to some aspects of the problem; the present work gives a more comprehensive treatment.

In what follows our main purpose is to answer the question: is 'scaling' valid at cosmic ray energies? The phenomena are considered in terms of increasing energy and attention is directed particularly towards a derivation of the mean multiplicity of pions $\langle n_s \rangle$ produced at cosmic ray energies.

In most of the analysis we ignore the effect of kaons in view of evidence from both accelerator and cosmic ray experiments that the fraction of kaons produced is small.

2. Predictions of the scaling hypothesis

The main feature of the model relevant to the present work is the prediction of a logarithmic rise in mean multiplicity of secondaries as summarized in figure 1. The sources of the data are indicated in the caption. Where comparisons are made with cosmic ray data a

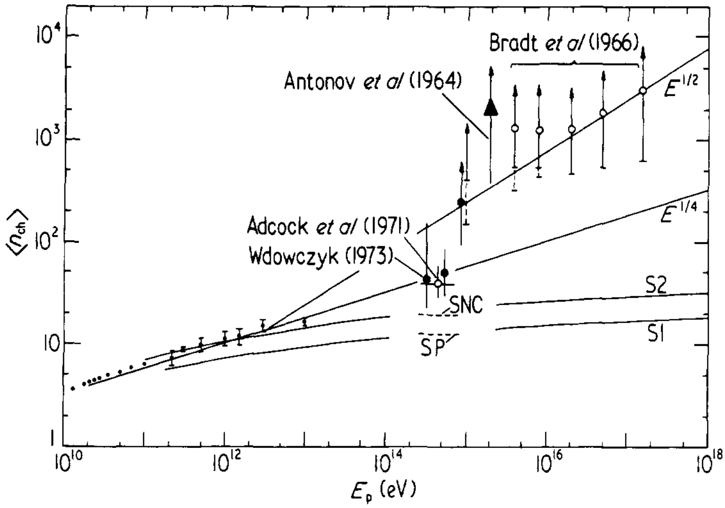

Figure 1. Mean multiplicity of charged secondaries against primary energy. Below 2×10^{12} eV the data refer to accelerator experiments on p–p collisions (summary by Jacob 1972). Between 2×10^{12} eV and 2×10^{13} eV they refer to hadron–emulsion nucleus interactions (Lohrman and Teucher 1962 and Gonguli and Malhotra 1972). At higher energies the multiplicities are weighted means for cosmic ray primary–air nucleus interactions.

The scaling 'law' S1 and S2 refer, respectively, to fits by Boggild *et al* (1971) to accelerator data at 19 GeV and by Morrison (1972) to ISR data (5×10^{11}–$1{\cdot}5\times10^{12}$ eV). SP denotes scaling prediction for primary protons and SNC is that for the 'normal' composition (SP and SNC are approximate and are given for illustration only).

knowledge of the energy spectrum of the secondaries is of considerable relevance. In the case of the scaling, the differential cross section is given by

$$\frac{d^2\sigma}{dx\,dp_t}=\frac{f(x,p_t)}{\{x^2+(p_t^2+m^2)p_0^{-2}\}^{1/2}}$$

where p_0x is the longitudinal momentum of the pion, mass m, in the C-system (p_0 = maximum possible C-system momentum of pion). As has been remarked already, integration over all x yields $\langle n_s\rangle \propto \ln p_0$ that is $\langle n_s\rangle \propto \ln E_p$, where E_p is the proton momentum in the L-system. Integration over values of x beyond x_0, ($x_0^2=(p_t^2+m^2)p_0^{-2}$) however yields

$$\left(\frac{d^2\sigma}{dx\,dp_t}\right)_0 \simeq \int_{x_0}^{1}\frac{f(x,p_t)}{x}$$

and the multiplicity of these 'energetic' pions is therefore constant, independent of E_p. It is important to stress this feature in view of the fact that in the cosmic ray case the rapidly falling primary energy spectrum often causes heavily weighted quantities (eg multiplicities) to be operative. For example, if scaling is valid, observations on single secondary cosmic rays will yield an effective multiplicity that is independent of E_p (values of x above 0·1 are mostly important in this case). Indeed the effective multiplicity is invariably less than the total value when comparison is made with cosmic ray derived multiplicities.

Before examining the cosmic ray data in detail consideration must be given to the problem of the extent to which cascading within the nucleus causes a difference to be expected between p–p and p–air nucleus collisions.

3. Intranuclear cascading

The problem of intranuclear cascading is one which pervades the whole subject of comparison of accelerator and cosmic ray data and a detailed examination is necessary. From general arguments it would be expected that there should be an increase in overall multiplicity, some reduction for high x_L values and an increase in small x_L values and an increase in the value of $\langle p_t \rangle$ ($x_L = E_\pi/E_p$ in the L-system; for E_p above several 10^{10} eV and $x \gtrsim 0{\cdot}05$, $x_L \simeq x$).

Accelerator measurements with nuclei having $A > 1$ as targets are restricted to rather low energies at present ($< 7 \times 10^{10}$ eV) but it might be expected that the fractional increase of $\langle n_s \rangle$ would not increase rapidly with primary energy and these 'low' energy data are, therefore, of value. A useful set of measurements has been made by Eichten *et al* (1972) using $2{\cdot}4 \times 10^{10}$ eV protons and a variety of target nuclei, from Be to Pb, and these will be considered.

In view of single particles in cosmic rays referring to $x_L \simeq 0{\cdot}2$ (for muon charge ratios) comparison can first be made in this region. The production densities for negative pions of 5×10^9 eV produced in p–Be and p–Al interactions for $p_t = 400$ MeV/c, were found to be 0·223 and 0·227 respectively. Using these figures and assuming a linear relation between production density and atomic mass at small A values the difference between p–p and p–air nucleus collisions can be shown to be about 0·5%. An examination of the dependence of this quantity on x and p_t shows that it is everywhere small and this in turn indicates that cascading should have a negligible effect in the cosmic ray case, for primary energies of 2×10^{10} eV.

At higher energies, in the absence so far of comprehensive data from the accelerators at 7×10^{10} eV and 3×10^{11} eV, recourse must be made to cosmic ray data. A useful summary has been given recently by Subramanian *et al* (1972) and it is shown that the total multiplicity $\langle n_s \rangle$ varies with the atomic mass of the struck nucleus as A^n with $n \simeq 0{\cdot}1$ (the experimental data are derived from experiments with hydrogen, carbon, brass and nuclear emulsion 'targets'). The energies in question are above 3×10^{10} eV. It is shown that the small increase in multiplicity is confined to small x_L values and this is interpreted in terms of the fragmentation model as due to successive fragmentation of nucleons in the target. These results are consistent with the accelerator data referred to earlier.

An important series of measurements has been made by Gierula and Wolter (1971) using nucleons from fragmentations with energies of about 10^{12} eV in nuclear emulsions. The results can be interpreted as showing that only some 7–8% of the total energy lost by a nucleon is dissipated in further collisions inside the average emulsion nucleus. Again, the effect of cascading for air nuclei would be expected to be very small.

If the effect of cascading on multiplicity is small, there should also be little effect on the mean transverse momentum of the secondaries. Some measure of confirmation comes from the work of Berdzenishvili *et al* (1971 and private communication). In this experiment interactions in carbon and polythene were studied at $\langle E_p \rangle = 4{\cdot}5 \times 10^{11}$ eV and the mean value of p_t was found to be $(0{\cdot}30 \pm 0{\cdot}03)$ GeV/c, a value not far from that for p–p collisions.

The smallness of intranuclear cascading (particularly for the energetic particles in the 'forward cone') demonstrated above is a well known phenomenon and indeed one of considerable fundamental interest in its own right. In fact an important role may be played by cosmic ray studies in future in elucidating this problem in view of the limitation of colliding beam experiments to p–p interactions. The attitude adopted (Van Hove 1972, Gottfried 1972, Goldhaber 1972, and Feinberg 1972a, b) is that the secondaries produced in a collision with a nucleon inside a nucleus cannot escape sufficiently from one another to act as independent particles and the ensuing multiplication is thereby reduced. An equivalent way of looking at the problem is to consider a regeneration time T_r during which the nonequilibrium system of particles has a smaller cross section than normal (Zatsepin 1962 postulated a similar situation for a nucleon which had undergone an inelastic collision). Following Feinberg (1972a, b) the ratio of the regeneration time T_r to the collision time T_c for nucleon–nucleon collisions is $T_r/T_c \simeq 2/E_p$ where E_p is the laboratory energy in GeV; regeneration would thus be very small above several tens of GeV.

Another way of looking at the problem is to follow the fragmentation hypothesis, in which the energetic secondaries come from the primary particle raised to some sort of excited state, this state not being able to de-excite until the particle has left the nucleus.

The relevance of these remarks to the present work is that the cascading would not be expected to become more serious at higher energies, particularly for the important small group of forward cone particles having 'large' x_L. In what follows we adopt this argument and neglect cascading completely.

4. The energy spectrum of primary cosmic rays

The prerequisite for some of the studies of the nuclear physical aspects of cosmic rays is the energy spectrum of the primaries. Direct measurements have been made up to about 10^{12} eV and there is some rough agreement as to the magnitude of the intensity at energies below that value. With the exception of a few very approximate direct measurements only indirect analyses are possible at higher energies. At energies between about 10^{12} eV and 10^{14} eV the analyses that have been made have involved assumptions about the high energy interaction processes and are therefore not of value here but above about 10^{14} eV the derived spectrum comes largely from measurements on showers near their maximum development at mountain altitudes and is rather insensitive to details of the nuclear physics.

Insofar as the primary particles have different masses (they comprise protons, α particles, etc) the 'primary energy spectrum' must be qualified by reference to the energy in question, that is, whether it is energy per nucleus or energy per nucleon. Figure 2 shows various spectra in terms of energy per nucleon.

The line marked Adcock *et al* (1971) is the spectrum used by us in a variety of previous studies; it will be noted that it has a constant integral exponent ($\gamma_p = 1{\cdot}6$) over the whole range (the integral spectrum has the form $E^{-\gamma_p}$).

More recently it has become evident that the spectrum may not, in fact, have this simple form—this possibility arises from a new direct measurement at low energy and a reconsideration of EAS results in the region of 10^{15} eV. The new measurements are those of Ryan *et al* (1972), shown in figure 2. Data are available for protons and α particles and both appear to have the same exponent ($\gamma_p = 1{\cdot}75 \pm 0{\cdot}03$); the proton measurements extend to about 2×10^{12} eV.

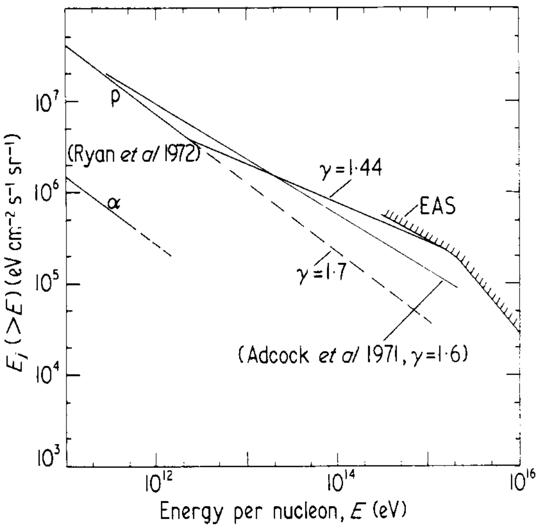

Figure 2. Energy spectra of primary cosmic rays. The values of γ are exponents of the integral primary spectrum.

Although measurements on extensive air showers near sea level have been used to give the form of the primary spectrum in the region of 10^{15} eV the best information comes from the data of the Chacaltaya experiment (Bradt *et al* 1966) where showers were detected near the maximum of their development. The intensity, in terms of energy per nucleon, cannot be determined accurately because of lack of knowledge of the mass composition but it is most unlikely that it is less than the value calculated for proton primaries by Hillas (1972) shown in figure 2. It is immediately seen that the extrapolation of the spectrum of Ryan *et al* is too low in the region of 10^{15} eV and that there is thus some evidence for a flattening in the range 10^{12}–10^{15} eV; joining the proton intensity at 2×10^{12} eV to the lower limit (proton) intensity at 10^{15} eV gives a spectrum with integral exponent $\gamma_p = 1{\cdot}44$. It should be noted that the processes of galactic diffusion and the nature of likely cosmic ray sources are of such complexity that the slope reduction is by no means ruled out on astrophysical grounds. In what follows we examine the consequences of both our original spectrum and this new form (further consideration is given to the question of the mass composition later).

5. Measurements on single unassociated particles

5.1. Muon charge ratio

The most accurately measured cosmic ray component at ground level is the muon component and data are available for both the charge ratio (μ^+/μ^-) and the energy spectrum. The charge ratio measurements show very little variation over the range $10^{10} < E_\mu < 2\times10^{12}$ eV (Ayre *et al* 1972, Ashley *et al* 1971) and at first sight this seems to support the fragmentation idea rather strongly for the range of primary energy $10^{11} < E_p < 2\times10^{13}$ eV assuming that the primary composition is constant in this region. However, this problem needs careful examination. Consider first the primary energy region below 10^{12} eV; here the primary composition is known and the characteristics of p–p collisions can be derived from ISR measurements so that if assumptions are made about the character of p–n, n–n and n–p interactions firm predictions of the μ^+/μ^- ratio are possible. If the ideas of fragmentation are adopted in their entirety (the charge ratio of the energetic secondaries is determined by the sign of the projectile) then it is found that the predicted ratio is higher than that observed (Frazer *et al* 1972, Hume *et al* 1973). There is thus some evidence for at least a measure of inapplicability of complete scaling at these energies; this observation is not necessarily at variance with the accelerator results, which also show some features which do not accord with complete scaling (§ 1) and which, very significantly, show lower π^+/π^- ratios for p–light nucleus interactions than for p–p interactions (at $2{\cdot}4 \times 10^{10}$ eV).

Garaffo *et al* (1972) have suggested that the discrepancy in the muon charge ratio arises from intranuclear cascading. However, as has been pointed out in § 3, the $2{\cdot}4 \times 10^{10}$ eV data suggest that intranuclear cascading effects are very small, particularly for the important large x_L values and we argued that a strong energy dependence is not to be expected. It is conceivable, but doubtful, that there is enough cascading to account for the muon charge ratio results. It is more likely (Hume *et al* 1973) that the reason is that the π^+/π^- ratio for p–n interactions differs from that for p–p collisions; a conclusion that is immediately inconsistent with expectation for limiting fragmentation but is consistent with the accelerator data at $2{\cdot}4 \times 10^{10}$ eV.

Turning to primary energies above 10^{12} eV, uncertainties in the primary composition preclude firm conclusions but it seems unlikely that the scaling prediction for the shape of the pion energy spectrum is grossly out for the next decade. If the primary spectrum does indeed flatten slightly (§ 4) and if the composition is unchanged then a small reduction in the muon charge ratio should result, because the effective value of x will fall and, with it, the π^+/π^- ratio. Such a fall is not seen and there is thus a little evidence for a modest breakdown in scaling here.

It is interesting to turn the argument round and assume that scaling is roughly valid above 10^{12} eV. The constancy of μ^+/μ^- would then appear to invalidate the conclusions of Grigorov *et al* (1969, 1971) that above this energy the primary n/p ratio increases considerably.

5.2. Muon energy spectrum

The relevance of the muon spectrum is that from it can be derived the pion production spectrum and this can be compared with the primary spectrum; an advantage is that the primary mass composition is unimportant, only the energy per nucleon spectrum having relevance. Because of the importance of 'large' values of x, the primary nucleon and pion spectra will have the same exponent, if the scaling hypothesis is valid.

It has been known for many years from magnetic spectrograph measurements that for near vertical muons below 10^{11} eV the exponent (γ_π) of the integral pion production spectrum is in the region 1·6 to 1·7. For example, Hayman and Wolfendale (1962) gave $\gamma_\pi = 1{\cdot}64 \pm 0{\cdot}05$ for pions giving muons with $10^{10} < E_\mu < 3 \times 10^{11}$ eV, that is, for protons with $10^{11} < E_p < 3 \times 10^{12}$ eV. The near equality of γ_π with the value of $\gamma_p = 1{\cdot}75 \pm 0{\cdot}03$ (or $\gamma_p = 1{\cdot}6$ in our earlier work) in this energy range indicates a constant multiplicity and is thus consistent with the ISR results. It should be remarked however that the result is not inconsistent within the experimental errors with a multiplicity law of the form $E_p^{1/4}$ together with the CKP pion energy spectrum when allowance is made for a cut-off of energetic pions produced in individual interactions (Brooke *et al* 1964). In the work cited it was shown that a primary spectrum with $\gamma_p = 1{\cdot}58$ resulted from taking the sea level muon and proton data and an $E_p^{1/4}$ law.

At higher muon energies, measurements with magnetic spectrographs become imprecise and recourse is made to other techniques; to use the measured intensity of muons underground as a function of depth, together with an assumed range–energy relation, or to examine the production of electromagnetic bursts in local absorbers or in the atmosphere.

A number of workers have examined the depth–intensity curve. For example, Osborne *et al* (1964) used the data then available to determine the spectrum to about 7×10^{12} eV. The result was that the exponent was constant to about 3×10^{12} eV and then increased. However, a subsequent analysis by Kiraly and Wolfendale (1970) using later underground intensities, largely the measurements of Menon *et al* (1967) and more recent theoretical estimates of the rate of energy loss (Erlykin 1965, Kelner and Kotov 1968, Kokoulin and Petrukhin 1969) showed that a muon spectrum with constant exponent out to about 10^{13} eV was favoured. This spectrum (designated D-70 by Kiraly and Wolfendale) has $\gamma_\mu = 2{\cdot}6$ (and thus $\gamma_\pi = 1{\cdot}60$). More recently, Ng *et al* (1973) have updated the analysis and used the very recent calculations of range fluctuation factors by Kiraly *et al* (1972). Their muon spectrum is slightly steeper ($\gamma_\mu = 2{\cdot}67$). It is given in figure 3, where two points from the deepest underground measurements (Khrishnaswamy *et al* 1971, Meyer *et al* 1970) are also shown. It should be remarked that intensities measured in these experiments at oblique angles suggest that this spectrum continues towards 10^{14} eV with similar (perhaps slightly increased) slope.

The burst studies prior to 1971 have been summarized by Wolfendale (1971). There is a rather wide spread in intensities but if these are attributed to random errors then an

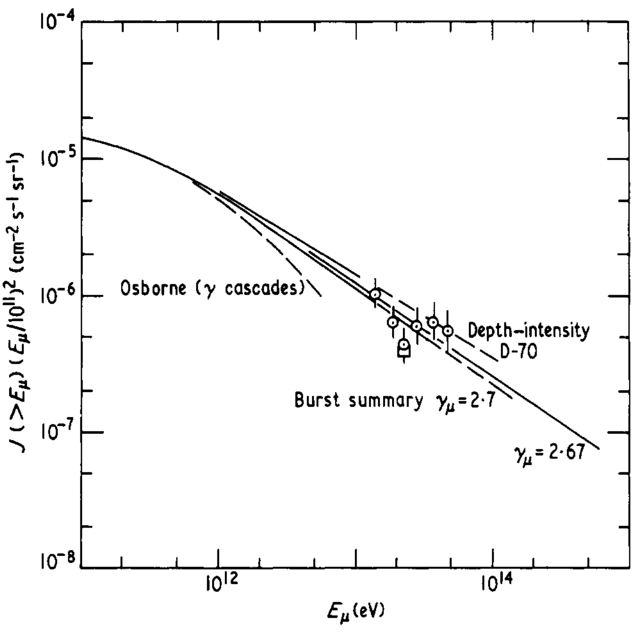

Figure 3. Muon energy spectra. The experimental points refer to measurements underground by Krishnaswamy *et al* (1971) (vertical and inclined measurements, designated by open circles) and by Meyer *et al* (1970) (vertical measurements only, designated by a square).

overall mean has significance. Figure 3 shows this summary spectrum; the measurements refer to the range $10^{12} < E_\mu < 10^{13}$ eV and the slope of the integral muon spectrum, $\gamma_\mu = 2{\cdot}7$, is reassuringly close to the value from the depth–intensity analysis.

Finally, an estimate of the spectrum of energetic muons can be made from the data on γ cascades in the atmosphere. Osborne and Wolfendale (1964) used the γ cascade data to protect the sea level spectrum under alternative assumptions about the nature of the parent particles, pions and kaons. They then used the muon spectrum derived from underground intensity measurements to determine the K/π ratio. We can now turn the argument round and estimate the muon spectrum, assuming that the K/π ratio continues at the value measured in the ISR experiment. The spectrum so derived is also shown in figure 3. Taking all the data together we conclude that the best estimate of the integral exponent is $\gamma_\mu = 2{\cdot}75 \pm 0{\cdot}1$ for $10^{12} < E_\mu < 4 \times 10^{13}$ eV.

The corresponding exponent of the integral pion spectrum is $\gamma_\pi = 1{\cdot}75 \pm 0{\cdot}1$ and this is compared with the two variants of the exponent of the primary spectrum in figure 4. It is apparent that if the primary spectrum does not have the flattening above 2×10^{12} eV referred to in § 4 then there is rough agreement with the scaling hypothesis. However, if the flattening is present there will be a divergence from the scaling prediction. New measurements of the primary spectrum will allow this question to be resolved.

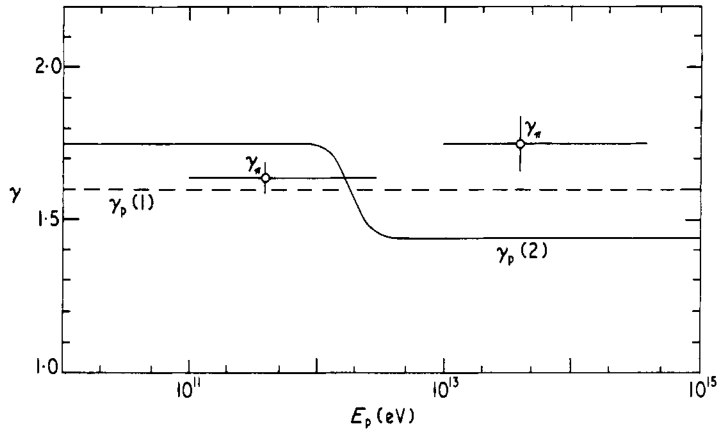

Figure 4. Comparison of exponents of pion production spectra and the primary spectrum. $\gamma_p(1)$ is the usual value taken for the primary exponent and $\gamma_p(2)$ is suggested by a combination of EAS data and the recent measurements of Ryan *et al* (1972).

6. Measurements on multiple muons

The multiplicity of secondaries produced by primaries of energy greater than those considered in the previous section can be examined by studying multiple muons which have individually high energies. The work of the Utah group (Cannon and Stenerson 1971) is unique in this respect and will be considered in some detail.

In the experiment, a very large detector is operated underground at such a depth that the muon threshold energy at ground level varies from about 6×10^{11} eV to 6×10^{12} eV, the actual value depending on zenith and azimuth angle. The data of Cannon and Stenerson (1971) have been analysed by Adcock *et al* (1971) under the assumption that the primary particles are mainly protons (see later). A comparison of the frequencies of detected double to triple muon events shows consistency with the $E^{1/4}$ multiplicity law, the mean primary energy in question being about (3–8) × 10^{14} eV. The virtue of comparing the frequencies of multiples is that the ratio is not very dependent on the value of the mean transverse momentum.

The mean multiplicities under these assumptions are shown in figure 1 and comparison can be made with the (approximate) mean multiplicity which would be expected from the scaling model (designated SP, ie scaling and primary protons). This multiplicity is smaller than that indicated as S2 because the very small x_L values in S2 do not contribute at all to the detected muons.

Mention has been made of the primary mass at the energies involved here (10^{14}–10^{16} eV). Examination of figure 2 indicates that, *a priori*, one would not expect much difference of composition from that in the directly measured region because there the composition did not appear to be changing with energy. Some confirmation for the view that protons still predominate comes from the reviews by Trumper (1970) and Thompson *et al* (1970). Amongst the evidence used, the strongest was probably that of Rappaport and Bradt (1969) in which the nuclear active particle energy flow at Mt Chacaltaya was measured. These workers found that the fluctuations in the ratio of energy flow to shower size was best explained if the composition of the primaries at 10^{15} eV were very similar to that at 10^{10} eV. Useful analyses by Chatterjee *et al* (1966, 1968) also gave the same result.

In fact, if the mass composition were the same as that at 10^{10} eV/nucleon the predicted rates of multiples would be somewhat higher than for the case of protons alone, the point being that the heavy nuclei although few in number are very efficient at producing multiple muons. Recently, Ormes and Balasubrahmanyan (1973) have given primary composition data for the energy range $3{\cdot}3 \times 10^{9}$–4×10^{10} eV/nucleon and we have recalculated the expected frequencies of multiples under the assumption that the composition is unchanged from 10^{10}–10^{15} eV/nucleon. The result is that there is a smaller discrepancy with the scaling prediction. An approximate representation is given in figure 1, where the effective multiplicity for scaling with which the 'measured' multiplicities are to be compared are indicated as SNC ('scaling' normal composition). A better way of comparing the data is given later in § 8. At this point it should be pointed out that it is possible that the composition is, in fact, purely protonic by 10^{14} eV, the heavy nuclei produced having fragmented near their sources although it should also be stated that the work of Ormes and Balasubrahmanyan (1973) gives an indication that the relative flux of iron nuclei is increasing rather than decreasing (at least at 10^{10} eV/nucleon).

Calculations of the expected frequencies of multiple muons in the Utah experiment (see figure 5) have recently been extended to cover the present situation (new data from Lowe 1973, private communication) in which the effective detector area is 80 m^2. Attention has been confined to a zenith angle of 60° for which geomagnetic effects are adjudged to be negligible. Coulomb scattering is neglected but range fluctuations for muons have been considered.

Preliminary results for the new, 80 m^2 data (for the case of primary protons) are given in figure 6, together with the predictions, and again it is seen that the data give a better fit to the $E_p^{1/4}$ law, than to the scaling predictions. The data have been grouped in three sets of muon threshold energy and the corresponding mean multiplicity derived; these are shown in figure 1 at the appropriate mean primary energies.

It should be pointed out that although a mean transverse momentum of 0·4 GeV/c has been adopted in the calculations (this being the expected 'scaling' value) the actual

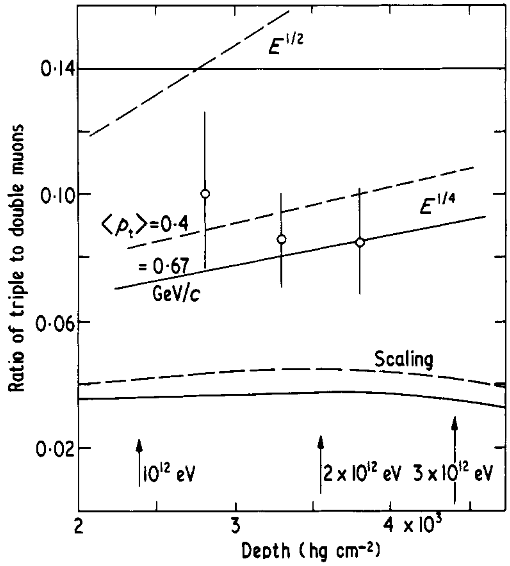

Figure 5. Ratio of triple to double muons as a function of depth (and thus muon threshold energy) in the experiment of Cannon and Stenerson (1971). The analysis for $\alpha = \frac{1}{4}$ and $\alpha = \frac{1}{2}$ was made by Adcock *et al* (1971); the predictions of scaling have been made by the present authors. The predictions refer to the situation where the primaries are mainly protons.

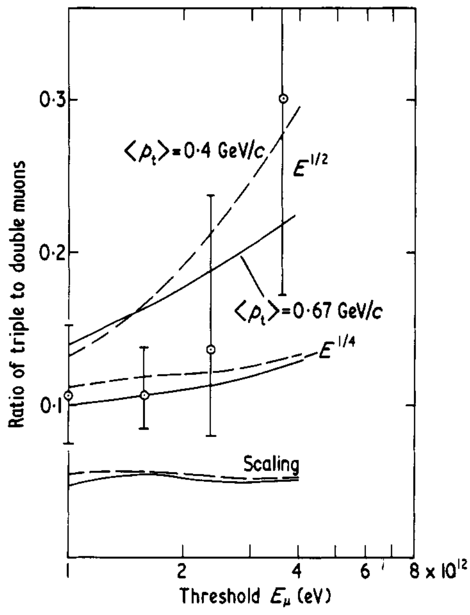

Figure 6. Ratio of triple to double muons as a function of muon threshold energy in the experiment of the Utah group (Lowe 1973, private communication). The predictions, which refer to the situation where the primaries are mainly protons, come from the work of Wdowczyk (1973).

experimental evidence favours a somewhat higher value. Thus, Adcock *et al* (1971), analysing earlier data from the Utah experiment by Coats *et al* (1970), derived $\langle p_t \rangle = 0{\cdot}6 \pm 0{\cdot}05$ GeV/c at $\langle E_p \rangle \simeq 2 \times 10^{14}$ eV. They also summarized the available data from other experiments and showed that there was some evidence for a slow rise in $\langle p_t \rangle$ with E_p. If this conclusion is true then a further prediction of scaling will have been invalidated and, furthermore, the mean multiplicities in figure 1 derived from the Utah work should be raised slightly.

7. Cosmic ray evidence on multiplicity above 10^{15} eV

7.1. Method of analysis

At primary energies above 10^{15} eV indirect information on nuclear interactions comes from studies of the electron–photon component of EAS. In particular, the position of the shower maximum (ie the depth in the atmosphere at which the shower has its greatest number of particles) can be used to give information about the multiplicity of secondary neutral pions. The pions produced in the first interaction of the primary and their ensuing γ rays are largely responsible for determining the position of the shower maximum and therefore, since EAS measurements can be made to at least 10^{18} eV, there is the possibility of determining the multiplicity of secondaries to such a high energy.

The multiplicity concerned is the total generated by the primary and one way of reconciling a high 'observed' multiplicity with the low multiplicity for p–p collisions according to the scaling hypothesis is to assume that the primaries are heavy nuclei. Wdowczyk and Wolfendale (1972) predicted the variation of effective primary mass A_{eff} with energy required to simulate the $E_p^{1/4}$ multiplicity law which appears to fit much of the high energy data, with the result shown in figure 7 (in fact, as will be seen shortly, the shower maximum data appears to indicate an even more rapid multiplicity increase and A_{eff} against E_p for an $E_p^{1/2}$ law is also given in figure 7). The A variation

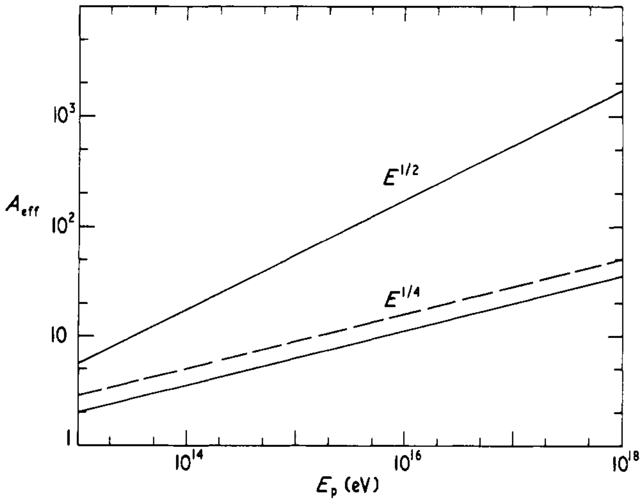

Figure 7. Variation of A_{eff} with incident nucleus energy. A_{eff} is that primary mass which with the scaling model gives a pion production spectrum close to that for the case of primary protons and an $E_p^{1/4}$ (or $E_p^{1/2}$) multiplicity law. The data for the scaling model has been taken from the work of Morrison (1972). The broken line refers to our earlier work (Wdowczyk and Wolfendale 1972) in which the form for scaling given by Boggild *et al* (1971) was used.

of figure 7 was put forward as a basis for discussion and not, of course, as a firm conclusion. In what follows we first examine the problem of evidence on the primary mass in this energy region.

7.2. Primary mass above 10^{15} eV

It was pointed out in § 6 that there was no evidence for a change in composition to the highest energies considered there ($\sim 10^{15}$ eV), that is, that the primaries appeared to be mainly protons. At higher energies, the evidence becomes less strong but there is still support for the primaries being mainly protons. Thus, Catz *et al* (1972, private communication) present evidence for this conclusion at energies in the range 10^{15}–10^{16} eV and Khristiansen *et al* (1971) likewise in the range 10^{15}–10^{17} eV; in both cases the evidence comes from studies of the fluctuations in shower age.

There is also information from measurements of the fluctuations of total number of electrons in showers having the same number of muons. Briefly, the fluctuations will be large for proton primaries and small for incident heavy nuclei. Adcock *et al* (1968) have examined the experimental data relating to this feature, and its interpretation, and it can be seen from their analysis that the Moscow data of Vernov *et al* (1967) indicate that the fluctuations are bigger than would be expected for pure heavy nuclei at about 10^{17} eV.

Another approach to the problem of primary mass comes from general astrophysical arguments. The existence of the rapid change of slope of the primary spectrum was mentioned in § 1 and this feature is presumably due either to a change in the character of the propagation of the particles in the galaxy or to a property of the sources (or to a mixture of both). Concerning the propagation change, Peters (1961) suggested that the change of slope might be due to an increase in diffusion coefficient which occurs when the irregularities in galactic magnetic field (thought of schematically as irregularly orientated 'clouds' of field) cease to scatter the particles isotropically. For all components of the radiation (protons, α's...) the exponent of the rigidity spectrum will increase by two above the same point but when the energy spectra are considered the displacement of the change point to higher energies for the higher masses causes the summed intensity to fall off more slowly and to be more in accord with experiment. The problems with this argument are that the transitional region near the change point is sharper than would be expected, and the exponent will increase by two above the pre-10^{15} eV value after the final component (iron) is encountered. The iron component will fall off above 10^{17} eV so that there is the measured region from 10^{17} eV to near 10^{20} eV, which has the same exponent as that from 10^{15} eV to 10^{17} eV, to be explained.

It must be concluded, that it is most unlikely that the simple diffusive model outlined above in which iron would preponderate around 10^{17} eV is correct. If the relative contributions of the various nuclei at the sources change with energy then high $\bar{A}$ values could, in principle, occur at energies above 10^{17} eV, and indeed it has been pointed out (eg by Karakula *et al* 1972) that problems of origin would be easier if this were true. There is in fact just the possibility that nuclei more massive than iron predominate beyond 10^{17} eV and preserve the constant spectral slope. However, their energy spectra would have to be dramatically flat in order to give agreement with contemporary total flux measurements and this explanation seems improbable.

In the next section we assume that the 'normal composition' prevails and return to the possibility of a composition change later.

7.3. Longitudinal development of EAS

As mentioned in § 7.1 the position of shower maximum is determined very largely by the multiplicity of pions produced in the first interaction. Figure 8 shows the expected dependence of height of maximum on primary energy for the case of proton primaries for three multiplicity 'laws': $E_p^{1/4}$, $E_p^{1/2}$ above 3×10^{12} eV, and $\ln E_p$. Although the calculations refer to protons there would be a quite negligible difference if the 'normal composition' were adopted instead because the shape of the individual longitudinal development curves is such that a large fraction of heavy nuclei is required in order to change the depth of maximum appreciably. For each multiplicity law, the energy spectrum is assumed to follow the CKP exponential form. The lines shown are from our own calculations. It should be pointed out that the $E^{1/4}$ and $E^{1/2}$ curves are close to the results of calculations by Turver (1973, private communication) at $E_p = 10^{17}$ eV and the $E^{1/4}$ value at 2×10^{15} eV is close to that given by Kalmykov *et al* (1971). The depth for the scaling law at 10^{17} eV is also close to the prediction of Turver.

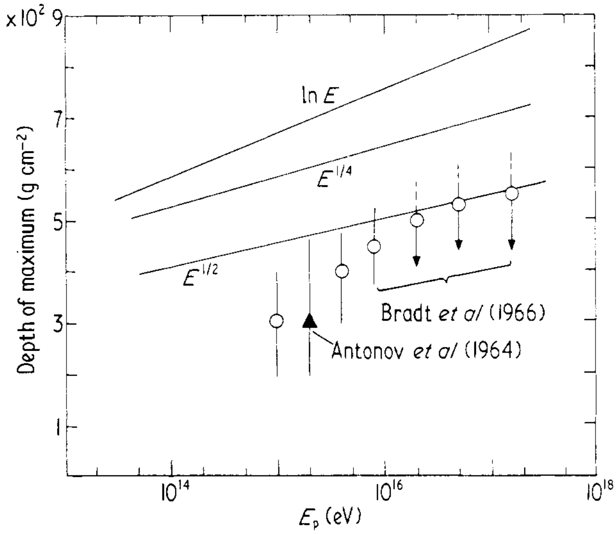

Figure 8. Dependence of depth of maximum development of EAS on primary energy for various multiplicity laws. The predictions refer to the case of primary protons.

Turning to the experimental results the most comprehensive measurements appear to be those of Bradt *et al* (1966). In this work EAS were recorded at various zenith angles at Mt Chacaltaya (vertical atmospheric depth 530 g cm^{-2}) and the authors used the data to give longitudinal development curves for showers of various energies. We have added data from the work of Antonov *et al* (1971) at aeroplane altitudes to help locate the maxima. The result of an earlier experiment by Antonov *et al* (1964) in which the shower maximum was determined in that experiment alone is also given in figure 8.

The experimental points in figure 8 together with the expected values have been used to determine the effective multiplicity in the first interaction as a function of energy and the results are given in figure 1. The relevant comparison for scaling is, approximately, with the curve marked S1.

8. Comparison with other analyses

Reference to figure 1 shows that the mean multiplicities above 10^{13} eV are higher than those based on the scaling hypothesis if the various steps in the argument are valid. Of these, the most important is the assertion that the effective mass of the primary nuclei is not much bigger than unity. Also important is our disregard of the effects of intranuclear cascading; if the difference between the derived mean multiplicities and those for scaling is due to cascading, however, it is evident that we are dealing with the onset of a process which is of remarkable magnitude and interest. In fact, the process would appear to invalidate the fragmentation hypothesis insofar as the continuing excited nucleon would need to de-excite within the nucleus, and, as was remarked in § 3, the experimental evidence does not favour any cascading effects for the important forward cone particles.

At this stage it is relevant to point out that other analyses have also led to the conclusion that the multiplicity rises more rapidly than given by the logarithmic law and, further, more rapidly than $E_p^{1/4}$. Kalmykov *et al* (1971) have analysed a variety of EAS data, including the energy flux per EAS particle, the attenuation length of showers between mountain altitudes and ground level and the energy spectrum of nuclear active particles and have concluded that a rapid rise of multiplicity is required above 10^{13} eV ($E_p^{1/2}$ gives a good fit). Furthermore, they argue for an increase in nucleon inelasticity above 10^{13} eV.

Many experiments have also been carried out to determine the lateral distribution of muons in EAS and most have shown that the mean width of the distribution is somewhat greater than expected on the basis of an $E_p^{1/4}$ law and the 'normal' mean transverse momentum. Although the measurements are difficult to make, being beset by problems of accurate shower axis location, and their interpretation is difficult because of a variety of effects (Coulomb scattering, geomagnetic deflexion, etc) they do appear to indicate mean heights of production somewhat greater than expected from scaling; the results are not inconsistent with an $E_p^{1/4}$ law.

Very recently, there has come evidence from the large EAS array at Yakutsk (Egorov *et al* 1971) in which the lateral distribution of Čerenkov light has been measured, to show that the electromagnetic component of EAS develops very rapidly in the atmosphere.

Finally, it should be remarked that Gaisser and Maurer (1973) have also examined EAS results in an endeavour to see whether scaling is valid. They, too, draw attention to the problem of intranuclear cascading but point out that, if this is small, then the measured μ/e ratio in showers near ground level indicates a power law increase of multiplicity with energy rather than the logarithmic variation.

9. Discussion and conclusions

9.1. *Scope of the discussion*

As stated in the Introduction, the main object of the work is to compare the multiplicities predicted from the scaling hypothesis with those derived from the various cosmic ray measurements. A variety of factors work to make an accurate comparison difficult, notably fluctuations in multiplicity (which displace the cosmic ray 'mean') and the use of different pion energy spectra in the two cases. Of greater importance, however, is the question of the primary mass composition. In what follows a comparison is made for two circumstances: mainly protons above 10^{13} eV and a composition with an increasing fraction of heavy nuclei.

9.2. Comparison of multiplicities if primaries are mainly protons

As has been pointed out already, this situation for the primary composition seems to fit the cosmic ray data best.

If this composition is correct and if intranuclear cascading is negligible the conclusions are as follows.

(i) $E_p < 3 \times 10^{12}$ eV. The single muon spectrum roughly confirms the applicability of the scaling expression but the charge ratio suggests that the fragmentation hypothesis is not completely valid.

(ii) $3 \times 10^{12} < E_p < 10^{14}$ eV. If the spectral slope is constant from 10^{10}–10^{15} eV then there is some measure of confirmation of scaling to about 10^{14} eV. However, if the steep slope determined by Ryan *et al* (1972) is correct then scaling is already breaking down here. The evidence seems to favour a breakdown.

(iii) $E_p > 10^{14}$ eV. The data suggest that the multiplicities diverge increasingly from the scaling prediction as the energy increases.

9.3. Comparison of multiplicities if primary mass increases considerably

Although many arguments have been put forward against a big increase in primary mass none is perfect and it is necessary to see whether scaling could be retrieved by a possible mass variation.

Figure 9 represents the effective mass necessary to give the experimental results of the Utah group, Antonov *et al* (1964) and the Chacaltaya group. The experimental values from figure 1 have been combined and used for figure 9.

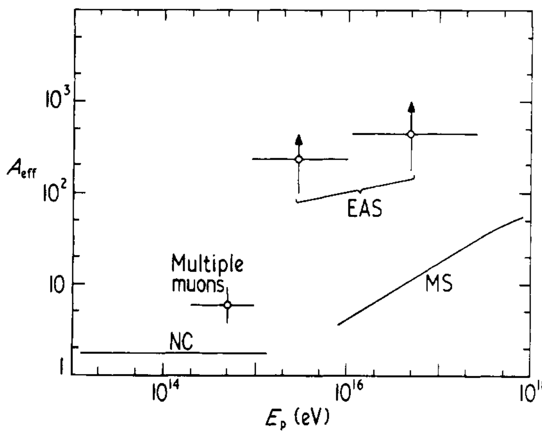

Figure 9. Effective primary mass against primary energy. NC denotes 'normal composition' and MS denotes 'modulated spectrum'. The values of A_{eff} are those required, together with scaling in individual p–nucleon interactions, to give the observed data: multiple muons in the range 10^{14}–2×10^{15} eV, particularly the observed ratios of 3's/2's and the observed longitudinal development of EAS above 10^{15} eV. The experimental data from figure 1 have been grouped together to give the experimental values of A_{eff}.

Also shown is the effective mass from the spectra of Ormes and Balasubrahmanyan (1973)—the 'normal composition': $A_{eff} = 1{\cdot}8$. The (unlikely) modulated spectrum of § 7.2 gives the line indicated MS. It can be seen that the value of A_{eff} required for scaling to be valid is still appreciably above the values expected fc r conventional modulation. Thus, unless the experimental data at high altitudes are inc orrect, it must be concluded that, either: (i) scaling is not valid above 10^{14} eV; or (ii) the effective primary mass increases rapidly with energy, being about 7 at $E_p = 10^{15}$ eV and about 200 at $E_p = 10^{17}$ eV. Furthermore, the process of fragmentation in the atmosphere must be such as to give rise to large fluctuations in overall development.

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